Problem Set 8

Prof. Oke

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *11.19.2025*

Due Friday, November 26, 2025 at 11:59 PM as PDF uploaded on Canvas. ***Show as much work as possible in order to get FULL credit.*** *There are 10 problems with a total of 63 points available.*

Problem 1 *Chi-square hypothesis test conclusions (9 points)*

What conclusions would be appropriate for an upper-tailed chi-square test in each of the following situations (where χ^2 is the test statistic)? (In each case, show explicitly how you compute and compare the critical value $\chi^2_{1-\alpha,\nu}$. Then circle the correct option (**i.**) or (**ii.**).)

(a) $\alpha = 0.01$, $k = 3$, $\chi^2 = 8.54$ *[3]*

i. Fail to reject H_0

ii. Reject H_0

(b) $\alpha = 0.10$, $k = 2$, $\chi^2 = 4.36$ *[3]*

i. Fail to reject H_0

ii. Reject H_0

(c) $\alpha = 0.01$, $k = 6$, $\chi^2 = 10.20$ *[3]*

i. Fail to reject H_0

ii. Reject H_0

Problem 2 *p-value of chi-square statistic (6 points)*

Calculate the p-value for an upper-tailed chi-square test in each of the following situations (show the Python/calculator function you use in each case):

[2] **(a)** $\chi^2 = 13.0$, $k = 6$

[2] **(b)** $\chi^2 = 18.0$, $k = 9$

[2] **(c)** $\chi^2 = 5.0$, $k = 4$

Problem 3 *Upper confidence bound (3 points)*

The charge-to-tap time (min) for a carbon steel in one type of open hearth furnace was determined for each heat[a] in a sample of size 46, resulting in a sample mean time of 382.1 and a population standard deviation of 31.5. Calculate a 95% upper confidence bound for true average charge-to-tap time.

(RIGHT:) An open hearth furnace being tapped at a Swedish steel mill. Source: https://en.wikipedia.org/wiki/File:Tappning_av_martinugn.jpg

[a] A "heat" describes each batch in an open-hearth process for steel production. Read this article for more information: https://en.wikipedia.org/wiki/Open_hearth_furnace

Problem 4 *Confidence Intervals and Sample Size (7 points)*

The article "Evaluating Tunnel Kiln Performance" (*Amer. Ceramic Soc. Bull.*, Aug 1997: 59–63) gave the following summary information for fracture strengths (MPa) of $n = 169$ ceramic bars fired in a particular kiln: $\overline{x} = 89.10, s = 3.73$.

(RIGHT:) A tunnel kiln. Source: https://blog.therseruk.com/hubfs/Tunnel%20Kiln%20for%20Refractories%20in%20UK%2017-2.jpg

[4] **(a)** Calculate a [two-sided] confidence interval for true average fracture strength using a confidence level of 95%. Does it appear that true average fracture strength has been precisely estimated?

[3] **(b)** Suppose the investigators had believed a priori that the population standard deviation was about 4 MPa. Based on this supposition, how large a sample would have been required to estimate μ to within 0.5 MPa with 95% confidence?

Problem 5 *Confidence intervals (5 points)*

A 95% confidence interval for a population mean, μ, is given as $(18.985, 21.015)$. This confidence interval is based on a simple random sample of 36 observations. Assume that all conditions necessary for inference are satisfied. Using the t-distribution, calculate the

(a) Margin of error *[1]*

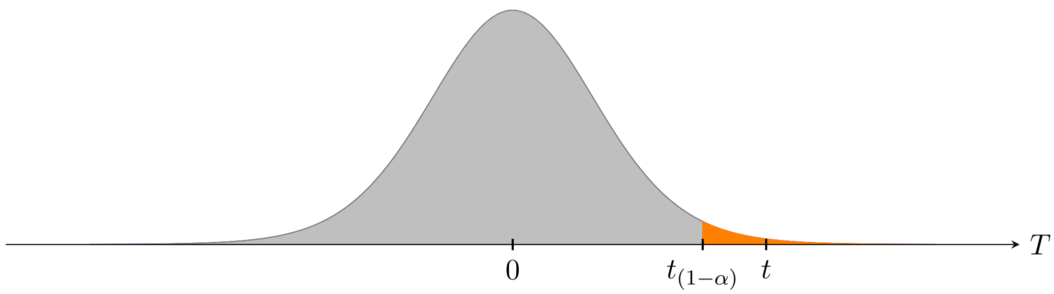
(b) Sample mean *[1]*

(c) Sample standard deviation *[3]*

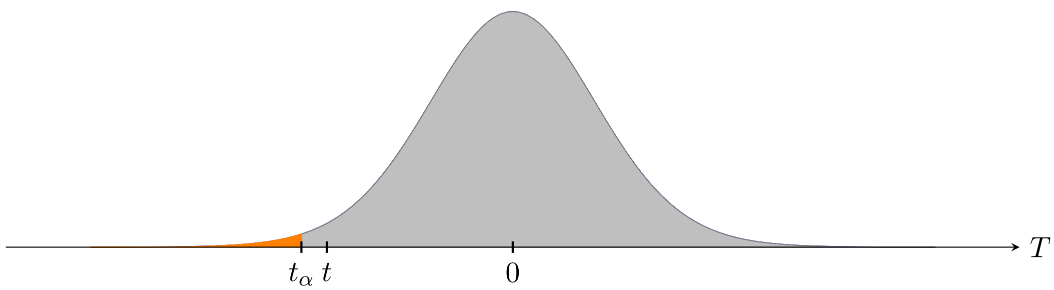
Problem 6 *Hypothesis testing (4 points)*

In the following hypothesis tests, decide whether to "Reject H_0" or "Fail to reject H_0" by comparing the Z or T scores (z or t, respectively) to the critical values. (Critical regions in orange.)

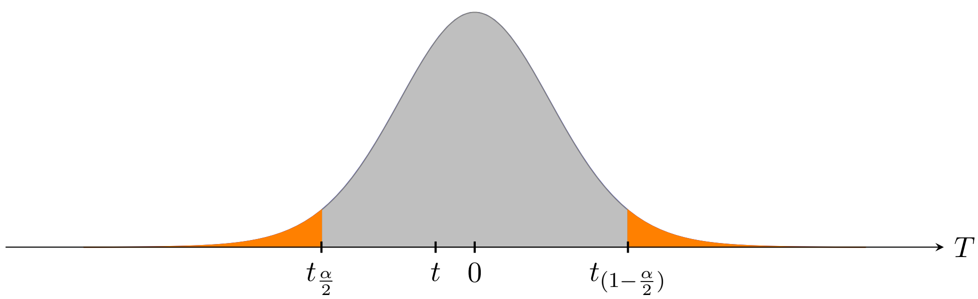
(a) $H_0 : \mu = \mu_0; H_1 : \mu > \mu_0$



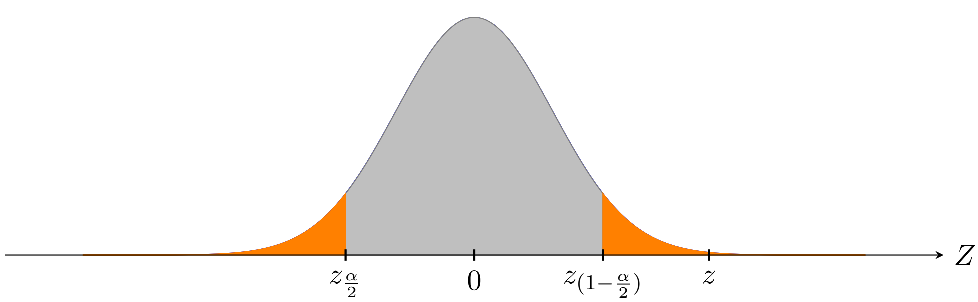
(b) $H_0 : \mu = \mu_0; H_1 : \mu < \mu_0$



(c) $H_0 : \mu = \mu_0; H_1 : \mu \neq \mu_0$



(d) $H_0 : \mu = \mu_0; H_1 : \mu \neq \mu_0$



Problem 7 *Identifying significance levels with Z-score (6 points)*

Let the test statistic Z have a standard normal distribution when H_0 is true. Find the significance level for each of the following situations (show your work and/or calculator/Matlab/Python functions):

(a) $H_1 : \mu > \mu_0$, critical region: $z \geq 1.88$.

(b) $H_1 : \mu < \mu_0$, critical region: $z \leq -2.75$.

(c) $H_1 : \mu \neq \mu_0$, critical region: $z \geq 2.88$ or $z \leq -2.88$.

Problem 8 *Identifying significance levels with T-score (6 points)*

Let the test statistic T have a t distribution when H_0 is true. Find the significance level for each of the following situations:

(a) $H_1 : \mu > \mu_0$, d.o.f. $= 15$, rejection region: $t \geq 3.733$.

(b) $H_1 : \mu < \mu_0$, d.o.f. $= 24$, rejection region: $t \leq -2.500$.

(c) $H_1 : \mu \neq \mu_0$, d.o.f. $= 31$, rejection region: $t \geq 1.697$ or $t \leq -1.697$.

Problem 9 *p-values (7 points)*

(a) Pairs of p-values and significance levels α are given. For each pair, state whether the observed p-value would lead to rejection of H_0 at the given significance level:

(i) p-value $= 0.084$; $\alpha = 0.05$ *[1]*

(ii) p-value $= 3.2 \times 10^{-5}$; $\alpha = 0.001$ *[1]*

(iii) p-value $= 0.039$; $\alpha = 0.01$ *[1]*

(b) Let μ denote the mean reaction time to a certain stimulus. For a large-sample test of $H_0 : \mu = 5$, versus $H_1 : \mu > 5$, find the p-value associated with the following test statistic values

(i) $z = 1.42$ *[2]*

(ii) $z = -0.11$ *[2]*

Problem 10 *Two-tailed hypothesis test (10 points)*

The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in $\overline{x} = 94.32$F and $s = 1.20$. Test $H_0 : \mu = 95$ versus $H_1 : \mu \neq 95$ using a two-tailed level 0.01 t test.

[2] **(a)** State the hypotheses

[3] **(b)** Compute the test statistic and critical value(s) OR p-value

[3] **(c)** Explicitly compare test statistic to the critical value (or p-value to α); sketch a supporting diagram of the distribution

[2] **(d)** State the outcome of the hypothesis test and write a concluding statement