Problem Set 7 **Prof. Oke**

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *11.4.2025*

Due Tuesday, November 11, 2025 at 1:00 PM as PDF uploaded on Canvas. *I strongly encourage you to write/type your responses directly on this document and upload it.* ***Show as much work as possible in order to get FULL credit.*** *There are 7 problems with a total of 64 points available.* ***Important:*** *If you use Python for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables, note this in the respective solution, as well.*

Problem 1 *(8 points)*

Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

(i) ☐ A sample of size $n = 100$ has a proportion parameter estimate $\hat{p} = 0.3$. If the sample observations are independent, then the sample satisfies the success-failure condition.

(ii) ☐ For a given confidence level, the margin of error decreases as the sample size increases.

(iii) ☐ If the 95% confidence interval for a proportion p is $(0.669, 0.731)$, then the margin of error is 0.062.

(iv) ☐ In computing a confidence interval, if the critical Z-score is 2.58, then the α corresponding to the confidence level of interest is 0.01.

(v) ☐ In a hypothesis test, the significance level α can also be understood as the probability of making a Type II error.

(vi) ☐ In hypothesis testing, failure to reject the null hypothesis H_0 does not mean the null hypothesis is necessarily true.

(vii) ☐ In a hypothesis test, if the p-value for the test statistic is 0.084 and $\alpha = 0.05$, then we would reject the null hypothesis.

(viii) ☐ In a hypothesis test, if the p-value for the test statistic is 3.2×10^{-5} and $\alpha = 0.01$, then we would reject the null hypothesis.

Problem 2 *Confidence interval of a proportion (10 points)*

A poll conducted in 2013 found that 52% of U.S. adult X (formerly Twitter) users get at least some news on X. The standard error for this estimate was 2.4%, and a normal distribution may be used to model the sample proportion $\hat{p}$. Now answer the following step-by-step questions in order to construct a 95% confidence interval for the fraction of U.S. adult X users who get some news on X.

[1] **(a)** State the value of $\hat{p}$.

[1] **(b)** Find the corresponding critical Z-score for a 95% confidence level. Show how you obtained your answer (wehther its via Python or another source.)

[2] **(c)** Compute the margin of error.

[2] **(d)** Write the 95% confidence interval of p.

[2] **(e)** Briefly interpret the interval in the context of this question.

[2] **(f)** Would you expect the 99% confidence interval to be narrower (smaller) or wider (larger)? Provide a reason for your answer.

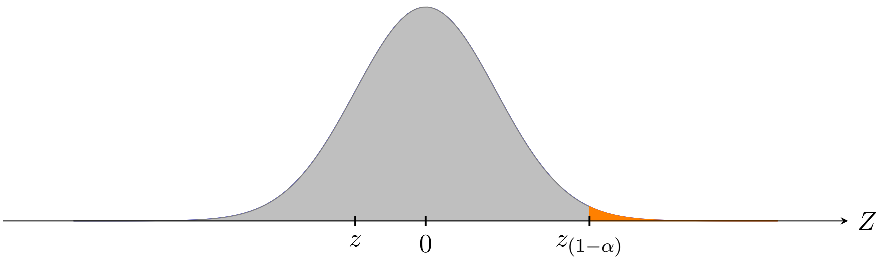
Problem 3 *Sample size (6 points)*

An article reports that when each football helmet in a random sample of 37 suspension-type helmets was subjected to a certain impact test, 24 showed damage. Let p denote the proportion of all helmets of this type that would show damage when tested in the prescribed manner. What sample size n would be required for the width of a 99% CI to be 0.1 (i.e. $ME = 0.05$)? (Points distribution: 3 points for test statistic calculation; 2 points for correct algebra in finding n; 1 point for correct final answer (to nearest whole number).) *[6]*

Problem 4 *Hypothesis testing (4 points)*

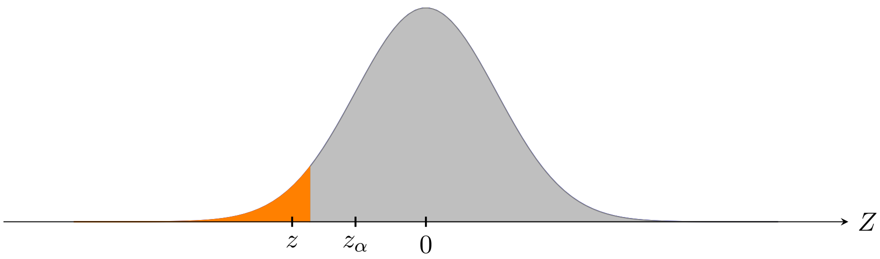
In the following hypothesis tests, circle the correct decision. Critical regions are shown in orange.

(a) $H_0 : p = p_0; H_1 : p > p_0$



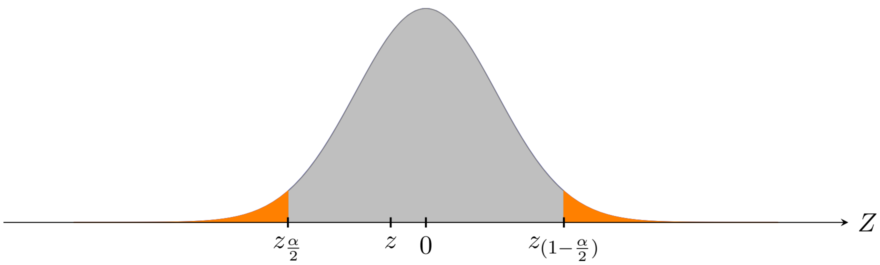
i. Reject H_0 **ii.** Fail to reject H_0

(b) $H_0 : p = p_0; H_1 : p < p_0$



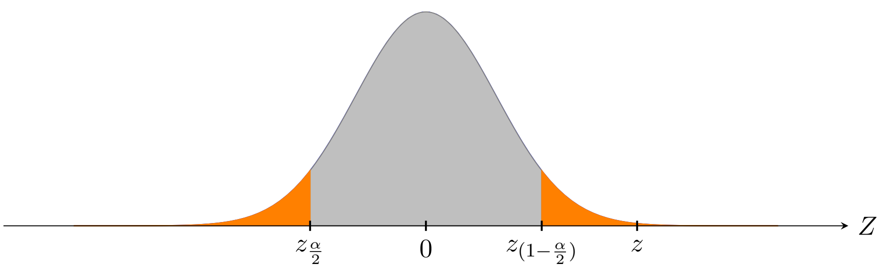
i. Reject H_0 **ii.** Fail to reject H_0

(c) $H_0 : p = p_0; H_1 : p \neq p_0$



i. Reject H_0 **ii.** Fail to reject H_0

(d) $H_0 : p = p_0; H_1 : p \neq p_0$



i. Reject H_0 **ii.** Fail to reject H_0

Problem 5 *Two-tailed hypothesis test using critical values (12 points)*

400 students were randomly sampled from a large university, and 289 said they did not get enough sleep. Conduct a hypothesis test to check whether this represents a statistically significant difference from 50%, and use a significance level of 0.05. In this problem, you are required to use the critical value approach (so no need to compute p-values). Your response will be graded on the following steps:

(a) State the hypotheses (there are two) *[2]*

(b) Compute test statistic *[2]*

(c) Find or state the critical values *[2]*

(d) Compare test statistic to critical values and sketch the standard normal distribution showing the critical values and the test statistic on the x-axis *[4]*

(e) State outcome of test and write concluding statement *[2]*

Problem 6 *Two-tailed hypothesis test using p-value (12 points)*

It is believed that nearsightedness affects about 8% of all children. In a random sample of 194 children, 21 are nearsighted. Conduct a hypothesis test ($\alpha = .05$) for the following question: do these data provide evidence that the 8% value is inaccurate? Your response will be graded on the following steps:

[2] **(a)** State the hypotheses (there are two)

[2] **(b)** Compute the test statistic

[2] **(c)** Find the p-value (show the Python line of code used to compute this)

[4] **(d)** Compare the appropriate values, and also include a sketch of the distribution showing the areas/regions

[2] **(e)** State outcome of test and write concluding statement

Problem 7 *Difference of two proportions testing using p-values (12 points)*

A quadcopter company is considering a new supplier for rotor blades. The new supplier would be more expensive, but they claim their higher-quality blades are more reliable, with 3% more blades passing inspection than their competitor ($\Delta_0 = 0.03$). The company's quality control engineer collects a sample of blades, examining 1000 blades from each supplier, and she finds that 899 blades pass inspection from the current supplier and 958 pass inspection from the prospective supplier. Using these data, set up and evaluate the hypotheses with a significance level $\alpha = .01$.

(a) State the hypotheses (this is a one-tailed test) *[2]*

(b) Find the standard error *[2]*

(c) Find the test statistic *[2]*

(d) Find the p-value *[2]*

(e) Compare the appropriate values *[1]*

(f) State the outcome of hypothesis test (reject/fail to reject H_0): *[1]*

(g) Write a final concluding statement in response to the question *[2]*