

**Due Tuesday, October 7, 2025 at 1:00 PM as PDF uploaded on Canvas.** Use this document as your template. **Show as much work as possible in order to get FULL credit.** There are 7 problems with a total of 41 points available. **Important:** If you use Python for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables or a calculator, note this in the respective solution, as well.

### Problem 1 (4 points)

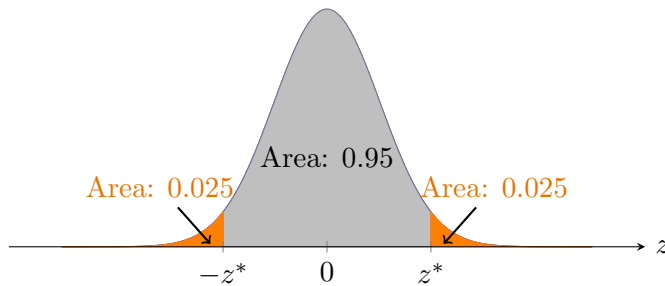
Respond “T” (*True*) or “F” (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

- (a)  If the natural logarithm of a random variable is normally distributed with parameters  $\mu$  and  $\sigma$ , then the variable is lognormally distributed with the same parameters.
- (b)  The mean of a lognormal distribution is equal to or greater than its median.
- (c)  The mean of a exponentially-distributed random variable is equal to the inverse of its variance.
- (d)  A binomial distribution with sufficiently large  $n$  can be approximated by a normal distribution with  $\mu = np$ .

## Problem 2: Normal and Lognormal Distributions (6 points)

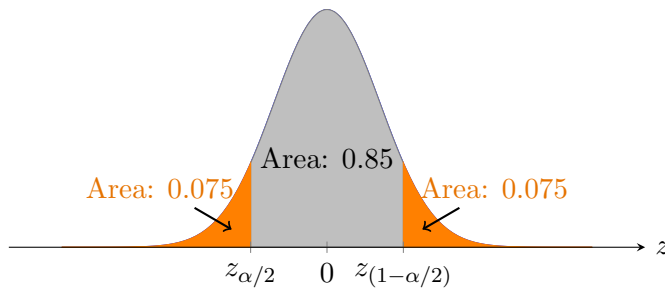
Choose the option that best fills in the blank.

- [1] (a) The figure below depicts the PDF of a standard normal distribution. What is the value of  $z^*$  in the figure?



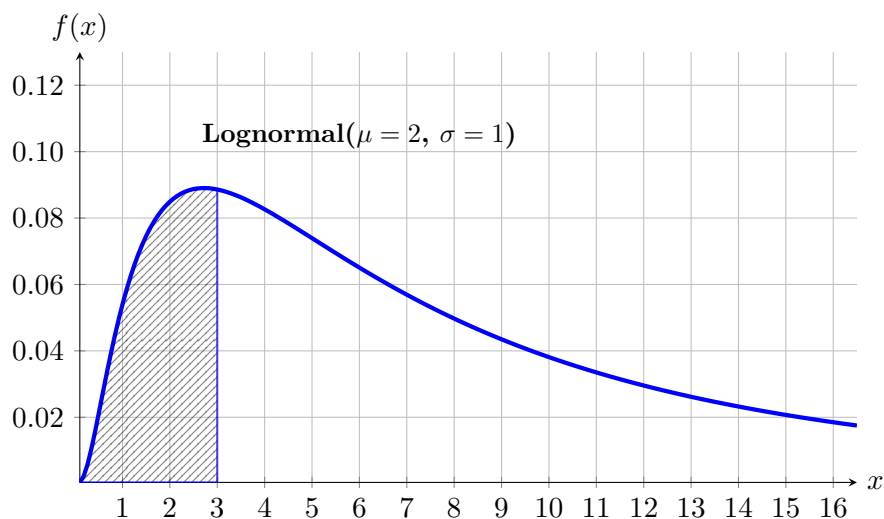
- (i) 0
- (ii) 1.65
- (iii) 1.96
- (iv) 2.58

- [1] (b) The figure below depicts the PDF of a standard normal distribution. What is the value of  $z_{\alpha/2}$  in the figure?



- (i) 0
- (ii) -1.04
- (iii) -1.28
- (iv) -1.44

- [2] (c) Find the area of the shaded portion in the figure below.



Answer:

**Problem 3: Lognormal Distribution (6 points)**

Given that the lifetime in days of an electronic component is lognormally distributed with  $\mu = 1.1$  and  $\sigma = 0.5$ .

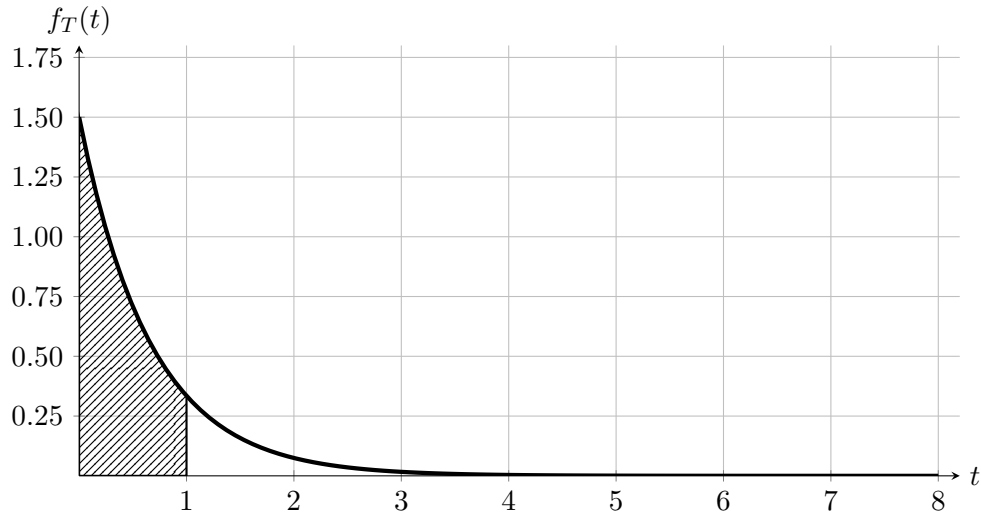
(a) Find the median lifetime of the component. [1]

(b) Find the mean lifetime of the component. [2]

(c) Find the probability that a component lasts between 3 and 5 days. [3]

**Problem 4: Exponential Distribution I (4 points)**

- [1] (a) The graph below is the PDF of an exponentially distributed random variable  $T$ , given by  $f_T(t) = \lambda e^{-\lambda t}$ . What is the value of the parameter  $\lambda$ ?



Answer:

- [1] (b) What is the mean of  $T$ ?

Answer:

- [2] (c) What is the probability represented by the shaded area in the figure in part (i)? (A numeric value is expected here, not just a symbolic expression.)

Answer:

**Problem 5: Exponential Distribution II** (*7 points*)

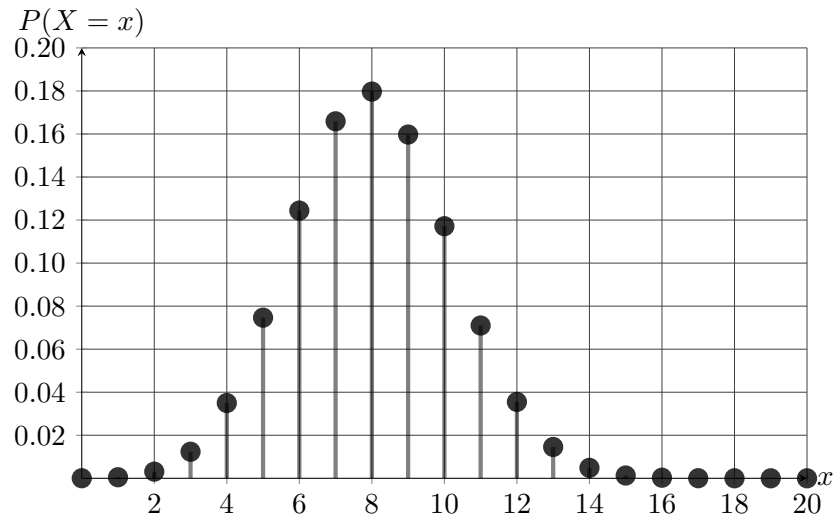
The delay time  $T$  of a flight is exponentially distributed with  $\lambda = 4$  (mean rate of occurrence per hour).

- (a) What is the expectation of  $T$ ? [1]
- (b) What is the standard deviation of  $T$ ? [1]
- (c) What is the probability that a flight is delayed by no more than half an hour? [2]
- (d) Given that a family member has already waited for half an hour, what is the probability that a certain flight will be further delayed by over an hour? [3]

### Problem 6: Binomial Distribution I (6 points)

Show brief amount of work for partial credit if answer is wrong. Not required however for full credit.

The PMF of a random variable  $X$  is given in the figure below.



- [1] (a) Use the figure to estimate the probability  $P(X = 8)$ .

Answer:

- [2] (b) Use the figure to estimate the probability  $P(X = 8 \cup X = 10)$ .

Answer:

- [2] (c) Use the figure to estimate the probability  $P(5 < X \leq 8)$ .

Answer:

- [1] (d) If the PMF in the figure above is that of a Binomial distribution with  $p = 0.4$ , what is  $\mathbb{E}(X)$ ?

Answer:

**Problem 7: Binomial Distribution II (8 points)**

75% of all vehicles examined at an emissions inspection station pass. Successive vehicles pass or fail independently of one another. Let  $X$  be the number of vehicles that pass the inspection out of the next  $n = 6$  vehicles inspected.

- (a) What is the expectation of  $X$ , i.e.  $\mathbb{E}[X]$ ? [1]
- (b) What is the standard deviation of  $X$ ? [1]
- (c) Find the probability that all of the next six vehicles inspected pass, i.e.  $P(X = 6)$ . [1]
- (d) Find the probability that only two of the next six vehicles inspected pass, i.e.  $P(X = 2)$ . [2]
- (e) Find the probability that at least four of the next six vehicles inspected pass. [3]