Problem Set 5 **Prof. Oke**

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *9.29.2025*

Due Tuesday, October 7, 2025 at 1:00 PM as PDF uploaded on Canvas. *Use this document as your template.* ***Show as much work as possible in order to get FULL credit.*** *There are 7 problems with a total of 41 points available.* ***Important:*** *If you use Python for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables or a calculator, note this in the respective solution, as well.*

Problem 1 *(4 points)*

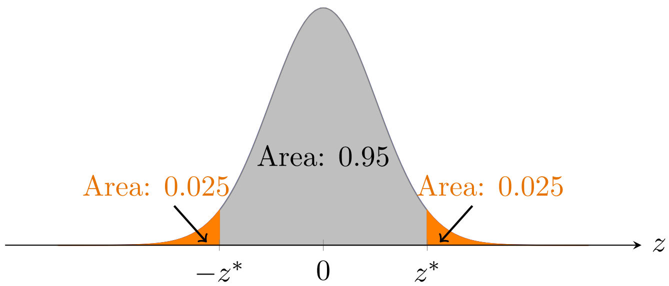
Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

	Answer	Statement
(a)	T	If the natural logarithm of a random variable is normally distributed with parameters μ and σ, then the variable is lognormally distributed with the same parameters.
(b)	F	The mean of a lognormal distribution is equal to or greater than its median.
(c)	F	The mean of a exponentially-distributed random variable is equal to the inverse of its variance.
(d)	T	A binomial distribution with sufficiently large n can be approximated by a normal distribution with $\mu = np$.

Problem 2: Normal and Lognormal Distributions *(6 points)*

Choose the option that best fills in the blank.

[1] **(a)** The figure below depicts the PDF of a standard normal distribution. What is the value of z^* in the figure?



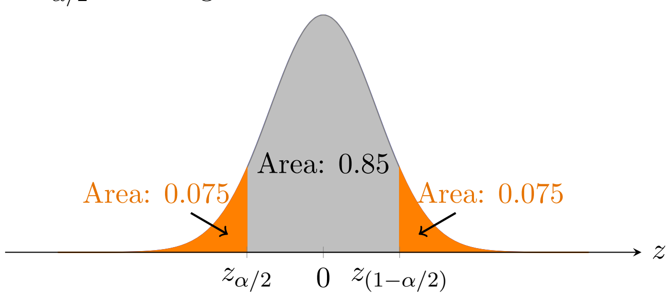
(i) 0

(ii) 1.65

(iii) 1.96

(iv) 2.58

[1] **(b)** The figure below depicts the PDF of a standard normal distribution. What is the value of $z_{\alpha/2}$ in the figure?



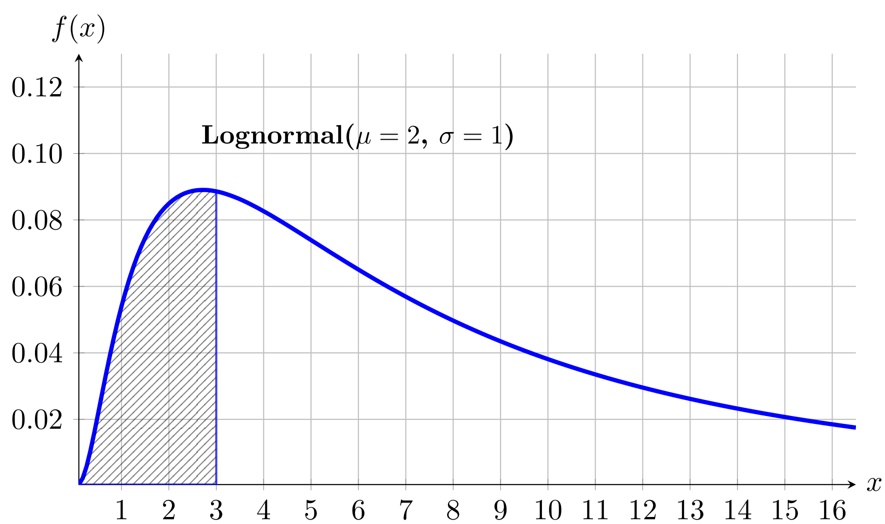
(i) 0

(ii) -1.04

(iii) -1.28

(iv) -1.44

[2] **(c)** Find the area of the shaded portion in the figure below.



Answer: $P(X \leq 3) = \Phi\left(\frac{\ln 3 - 2}{1}\right) = \Phi(-0.9014) \approx 0.1837 \approx 0.184$

Problem 3: Lognormal Distribution *(6 points)*

Given that the lifetime in days of an electronic component is lognormally distributed with $\mu = 1.1$ and $\sigma = 0.5$.

(a) Find the median lifetime of the component. *[1]*

$$\text{Median}(X) = e^{\mu}$$

Thus

$$\text{Median}(X) = e^{1.1} = 3.0042 \text{ days.}$$

(b) Find the mean lifetime of the component. *[2]*

$$\mathbb{E}[X] = e^{\mu+\sigma^2/2} = e^{1.1+0.5^2/2} = e^{1.225} = 3.4042 \text{ days.}$$

(c) Find the probability that a component lasts between 3 and 5 days. *[3]*

Let $Z = \dfrac{\ln X - \mu}{\sigma}$.
Then

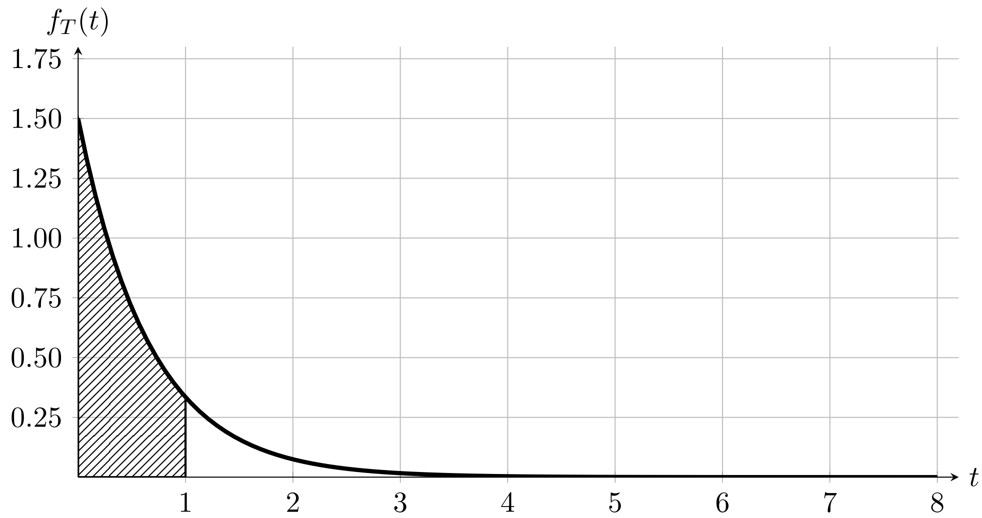
$$P(3 \leq X \leq 5) = \Phi\left(\frac{\ln 5 - 1.1}{0.5}\right) - \Phi\left(\frac{\ln 3 - 1.1}{0.5}\right).$$

Compute $z_1 = \dfrac{\ln 3 - 1.1}{0.5} = -0.0028$, $z_2 = \dfrac{\ln 5 - 1.1}{0.5} = 1.0189$. Therefore

$$P(3 \leq X \leq 5) = \Phi(1.0189) - \Phi(-0.0028) \approx 0.8460 - 0.4990 = 0.3470.$$

Problem 4: Exponential Distribution I *(4 points)*

[1] **(a)** The graph below is the PDF of an exponentially distributed random variable T, given by $f_T(t) = \lambda e^{-\lambda t}$. What is the value of the parameter λ?



Answer: $\lambda = f_T(0) = 1.5$

[1] **(b)** What is the mean of T?

Answer: $1/\lambda = \dfrac{1}{1.5} = 0.6667$ h

[2] **(c)** What is the probability represented by the shaded area in the figure in part **(i)**? (A numeric value is expected here, not just a symbolic expression.)

Answer: $P(0 \leq T \leq 1) = 1 - e^{-\lambda \cdot 1} = 1 - e^{-1.5} = 0.7769$

Problem 5: Exponential Distribution II *(7 points)*

The delay time T of a flight is exponentially distributed with $\lambda = 4$ (mean rate of occurrence per hour).

(a) What is the expectation of T? *[1]*

For $T \sim \text{Exp}(\lambda)$, $\mathbb{E}[T] = 1/\lambda$. With $\lambda = 4$,

$$\mathbb{E}[T] = \frac{1}{4} = 0.25 \text{ h} = 15 \text{ min}.$$

(b) What is the standard deviation of T? *[1]*

For an exponential, $\text{SD}(T) = 1/\lambda$. Hence

$$\text{SD}(T) = \frac{1}{4} = 0.25 \text{ h}.$$

(c) What is the probability that a flight is delayed by no more than half an hour? *[2]*

$$P(T \leq 0.5) = 1 - e^{-\lambda(0.5)} = 1 - e^{-2} = 0.8647.$$

(d) Given that a family member has already waited for half an hour, what is the probability that a certain flight will be further delayed by over an hour? *[3]*

Let T be the total delay time (in hours), and suppose that $T \sim \text{Exp}(\lambda)$ with $\lambda = 4$. We are asked to find

$$P(T > 1.5 \mid T > 0.5)$$

Using the memoryless property of the exponential distribution,

$$P(T > 1.5 \mid T > 0.5) = P(T > 1)$$

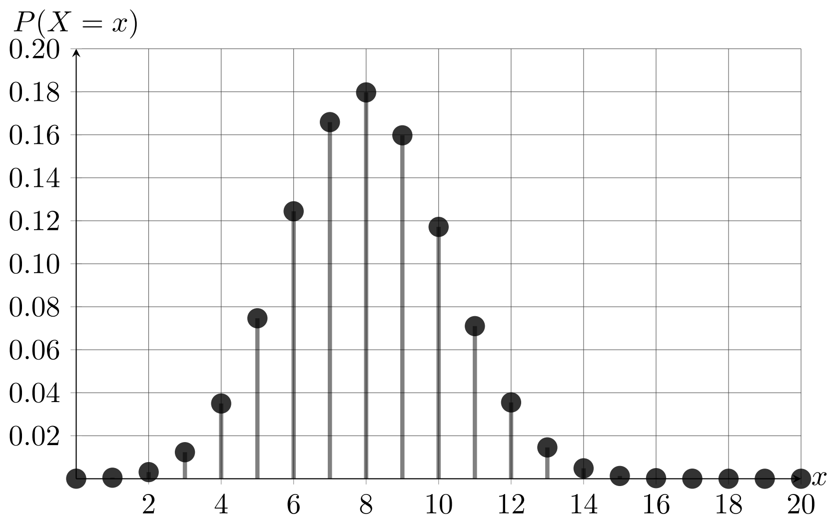
Then,

$$P(T > 1) = e^{-\lambda \cdot 1} = e^{-4} \approx 0.0183.$$

Problem 6: Binomial Distribution I *(6 points)*

Show brief amount of work for partial credit if answer is wrong. Not required however for full credit.

The PMF of a random variable X is given in the figure below.



[1] **(a)** Use the figure to estimate the probability $P(X = 8)$.

Answer: $P(X = 8) \approx 0.18$

[2] **(b)** Use the figure to estimate the probability $P(X = 8 \cup X = 10)$.

Answer: $P(X = 8 \cup X = 10) \approx 0.18 + 0.12 = 0.30$

[2] **(c)** Use the figure to estimate the probability $P(5 < X \leq 8)$.

Answer: $P(5 < X \leq 8) = P(6) + P(7) + P(8) \approx 0.47$

[1] **(d)** If the PMF in the figure above is that of a Binomial distribution with $p = 0.4$, what is $\mathbb{E}(X)$?

Answer: $\mathbb{E}(X) = np = 20(0.4) = 8$

Problem 7: Binomial Distribution II *(8 points)*

75% of all vehicles examined at an emissions inspection station pass. Successive vehicles pass or fail independently of one another. Let X be the number of vehicles that pass the inspection out of the next $n = 6$ vehicles inspected.

(a) What is the expectation of X, i.e. $\mathbb{E}[X]$? *[1]*

$X \sim \text{Bin}(n = 6, p = 0.75)$. Then

$$\mathbb{E}[X] = np = 6(0.75) = 4.5$$

(b) What is the standard deviation of X? *[1]*

$$\begin{aligned}\text{SD}(X) &= \sqrt{np(1-p)} \\ &= \sqrt{6(0.75)(0.25)} \\ &= \sqrt{1.125} = 1.0607\end{aligned}$$

(c) Find the probability that all of the next six vehicles inspected pass, i.e. $P(X = 6)$. *[1]*

$$\begin{aligned}P(X = 6) &= \binom{6}{6}(0.75)^6(0.25)^0 \\ &= (0.75)^6 \\ &= 0.1780\end{aligned}$$

(d) Find the probability that only two of the next six vehicles inspected pass, i.e. $P(X = 2)$. *[2]*

$$\begin{aligned}P(X = 2) &= \binom{6}{2}(0.75)^2(0.25)^4 \\ &= 15(0.5625)(0.00390625) \\ &= 0.0330\end{aligned}$$

(e) Find the probability that at least four of the next six vehicles inspected pass. *[3]*

$$\begin{aligned}P(X \geq 4) &= P(X = 4) + P(X = 5) + P(X = 6) \\ &= \binom{6}{4}(0.75)^4(0.25)^2 + \binom{6}{5}(0.75)^5(0.25) + (0.75)^6 \\ &= 0.8306\end{aligned}$$