Problem Set 4

Prof. Oke

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *9.23.2025*

Due September 30, 2025 at 11:59 PM as PDF uploaded via Gradescope. *For ease of grading, use this document as your template (either print/write/upload, use LaTeXor edit using a tablet device)* ***Show as much work as possible in order to get FULL credit.*** *There are 7 problems with a total of 30 points available.* ***Important:*** *If you use Python for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables, note this in the respective solution, as well.*

Problem 1 *(5 points)*

Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

(i) ☐ If Z represents the standard normal variable, then the mean of Z is 1.

(ii) ☐ To standardize a normally distributed random variable, we find the difference from its mean and divide the result by its standard deviation.

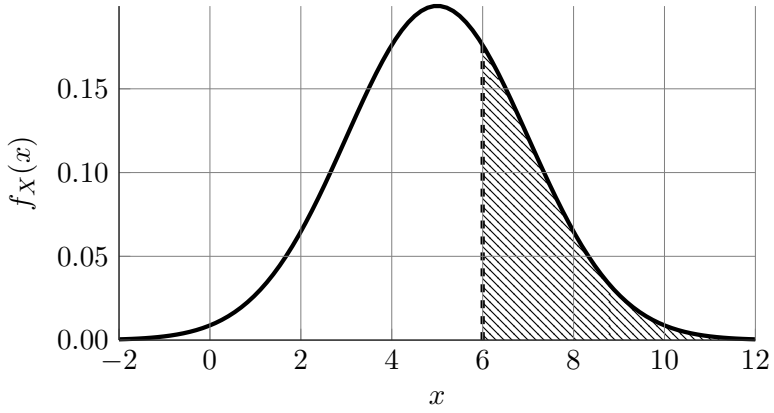
(iii) ☐ The lifetime of a lightbulb is normally distributed with $\mu = 1400$ hrs and $\sigma = 200$ hrs. The 20th percentile of the lifetimes is approximately 1140 hrs.

(iv) ☐ The lifetime of a lightbulb is normally distributed with $\mu = 1400$ hrs and $\sigma = 200$ hrs. The probability that the lightbulb will last more than 1700 hrs is 0.933.

(v) ☐ The area under a PDF can be less than or equal to 1.

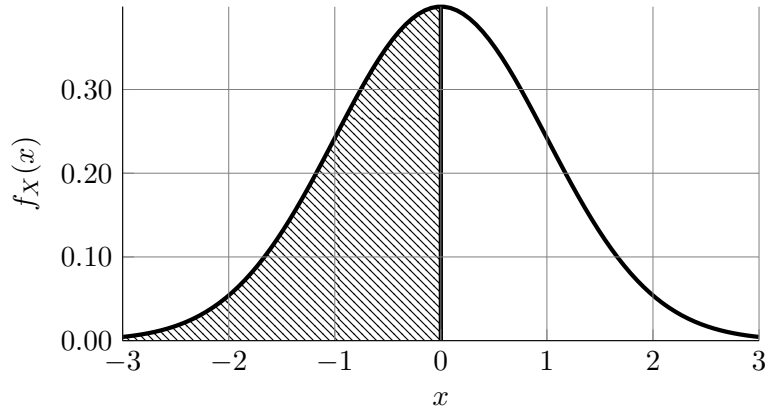
Problem 2 *(2 points)*

(a) Write down the expression of the probability represented by the shaded portion of the normal PDF below. For example, $P(X \leq 2)$. Note that a dashed vertical boundary indicates "$>$" or "$<$," while a solid vertical boundary indicates "$\geq$" or "$\leq$." *[1]*



Answer:

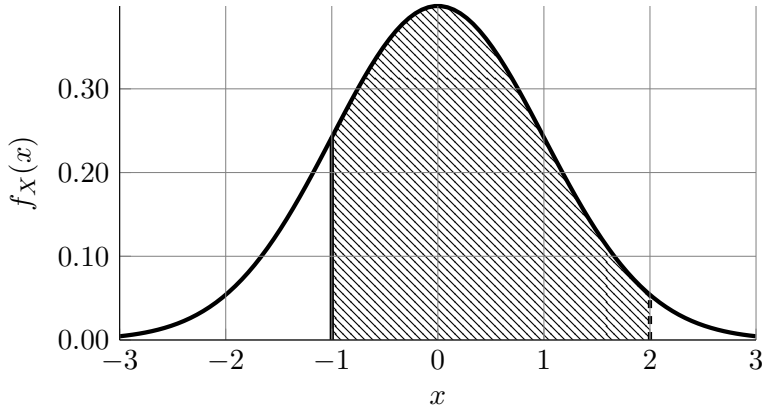
(b) Write down the expression of the probability represented by the shaded portion of the normal PDF below. For example, $P(X \leq 2)$. Note that a dashed vertical boundary indicates "$>$" or "$<$," while a solid vertical boundary indicates "$\geq$" or "$\leq$." *[1]*



Answer:

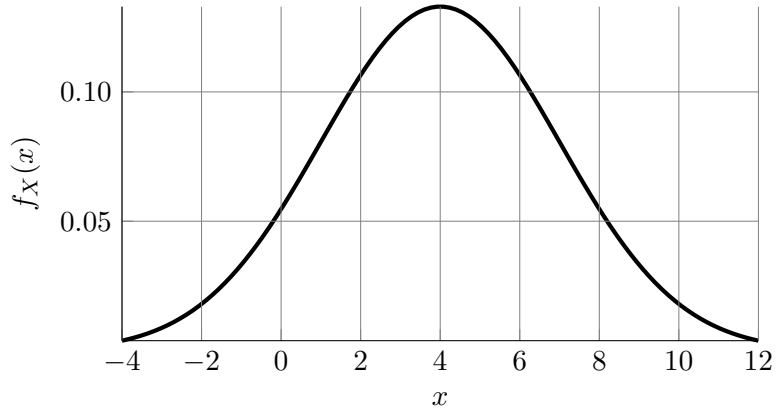
Problem 3 *(2 points)*

(a) Write down the expression of the probability represented by the shaded portion of the normal PDF below. For example, $P(X \leq 2)$. Note that a dashed vertical boundary indicates "$>$" or "$<$," while a solid vertical boundary indicates "$\geq$" or "$\leq$." *[1]*



Answer:

(b) Below is the PDF of a given normal distribution. What is the median of this distribution? *[1]*

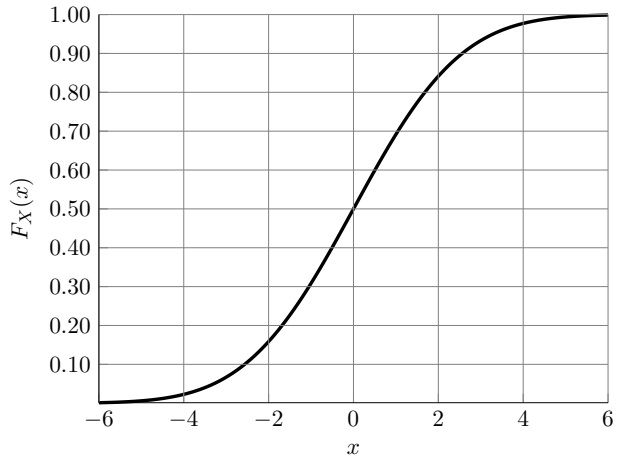


Answer:

Problem 4 *(4 points)*

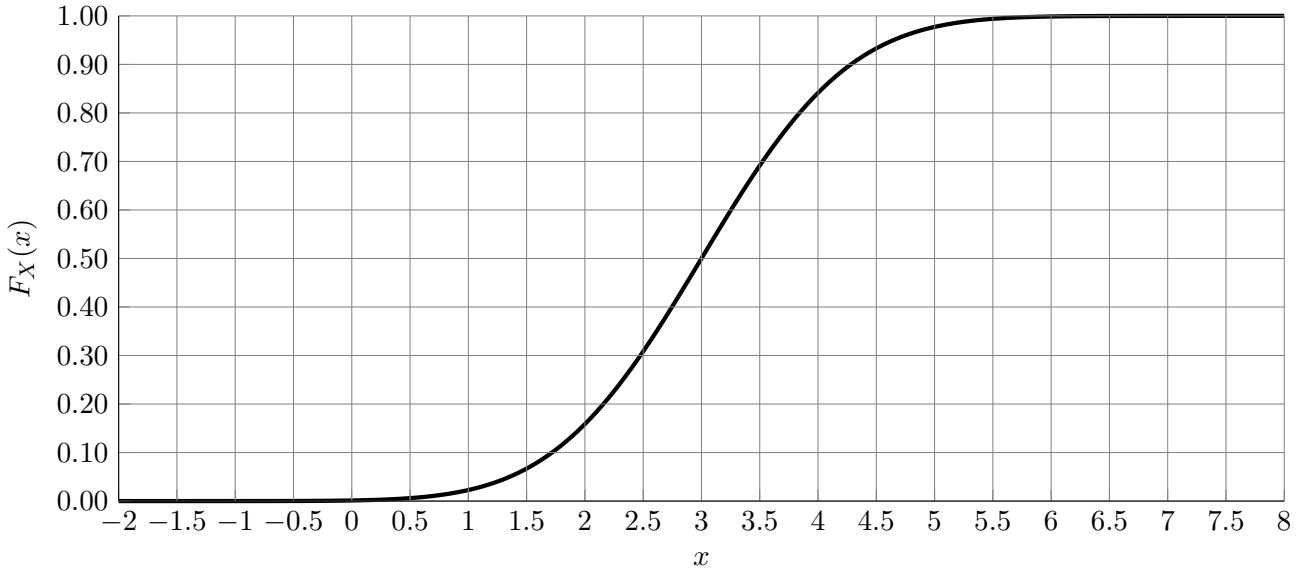
In the following problems, show how you arrive at the answer on the graph.

[2] **(a)** Below is the CDF of a given normal distribution. What is the mean of this distribution?



Answer:

[2] **(b)** Below is the CDF of a given normal distribution. Estimate the probability $P(X > 3.5)$.

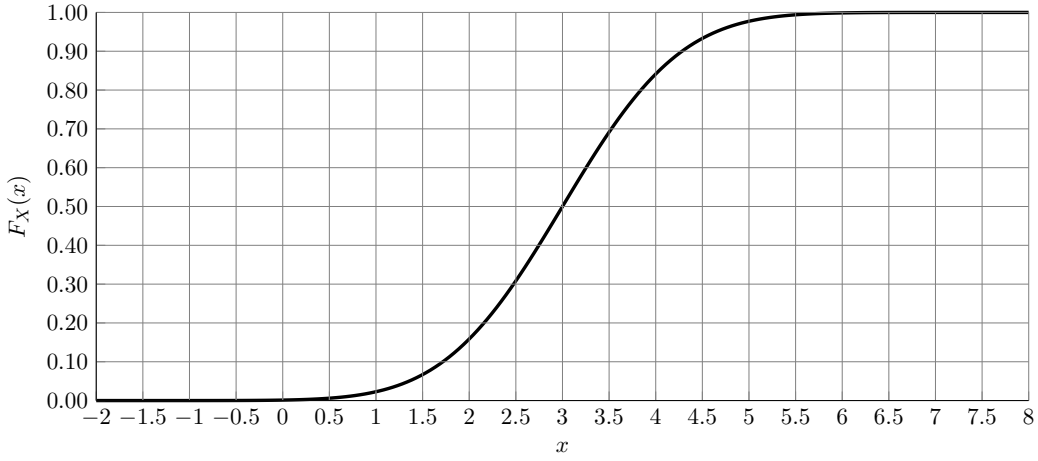


Answer:

Problem 5 *(4 points)*

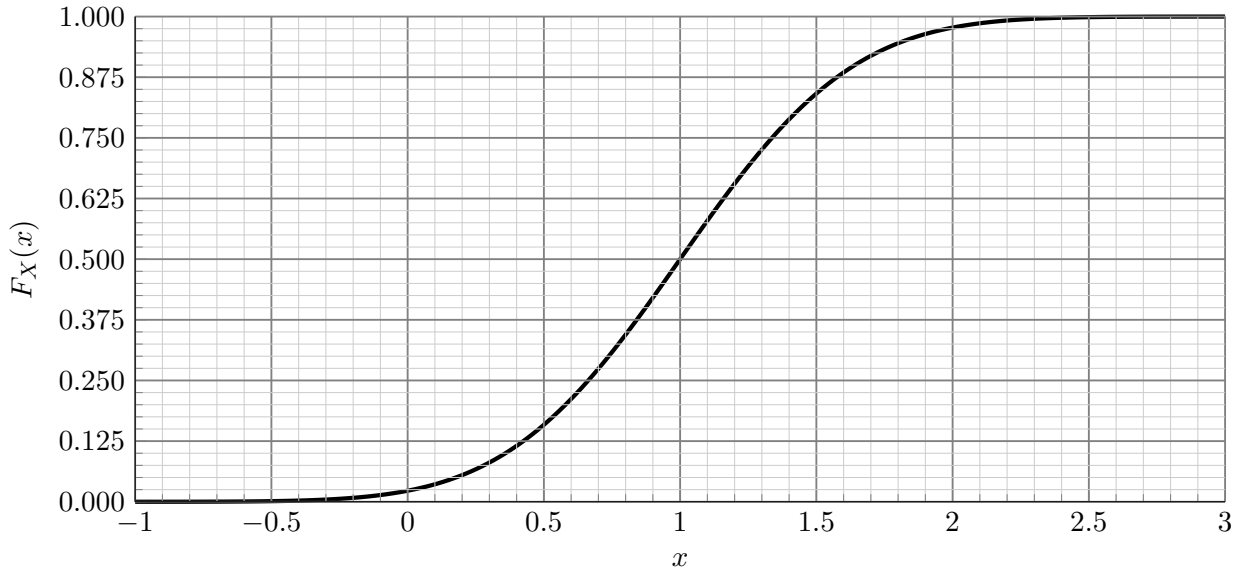
In the following problems, show how you arrive at the answer on the graph.

(a) Below is the CDF of a given normal distribution. Estimate the quantity $F_X^{-1}(0.3)$. *[2]*



Answer:

(b) Below is the CDF of a given normal distribution. Estimate the first quartile of the distribution. *[2]*



Answer:

Problem 6 *Standard Normal Distribution (8 points)*

What percent of a standard normal distribution $\mathcal{N}(\mu = 0, \sigma = 1)$ is found in each region? Sketch the accompanying curve along with your answer.

[2 pts] (a) $Z < -1.35$

[3 pts] (b) $-0.4 < Z < 1.5$

[3 pts] (c) $|Z| > 2$

Problem 7 *Normal Distribution (5 points)*

The average daily high temperature in June in LA is 77°F with a standard deviation of 5°F. Suppose that the temperatures in June closely follow a normal distribution.

(a) What is the probability of observing an 83°F temperature or higher in LA during a randomly chosen day in June? *[2]*

(b) How cool are the coldest 10% of the days (days with lowest average high temperature) during June in LA? *[3]*