Problem Set 4 **Prof. Oke**

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *09.23.2025*

Due October 8, 2024 at 11:59 PM as PDF uploaded via Gradescope. *If it helps and if possible, you can write your responses directly on this document and upload it instead.* ***Show as much work as possible in order to get FULL credit.*** *There are FOUR problems with a total of 27 points available.* ***Important:*** *If you use MATLAB/Python for any probability computations, briefly write/include the statements you used to arrive at your answers. If instead you use probability tables, note this in the respective solution, as well.*

Problem 1 *(5 points)*

Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

(i) F — If Z represents the standard normal variable, then the mean of Z is 1. The mean of Z is 0, and its standard deviation and variance are 1.

(ii) T — To standardize a normally distributed random variable, we find the difference from its mean and divide the result by its standard deviation. Yes. The $Z-score$ is given by $Z = \frac{X-\mu}{\sigma}$, where X is a normally distributed random variable and Z its standardized value.

(iii) F — The lifetime of a lightbulb is normally distributed with $\mu = 1400$ hrs and $\sigma = 200$ hrs. The 20th percentile of the lifetimes is approximately 1140 hrs. The 20th percentile is given by $F^{-1}(0.2) = 1231.7$ (can be found using `norminv(.2,1400,200)` in MATLAB).

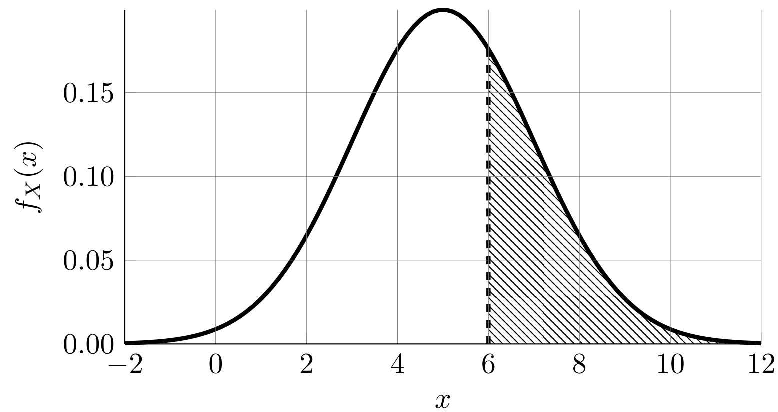
(iv) F — The lifetime of a lightbulb is normally distributed with $\mu = 1400$ hrs and $\sigma = 200$ hrs. The probability that the lightbulb will last more than 1700 hrs is 0.933. $P(X > 1700) = 1 - P(X \leq 1700) = 1 - \Phi\left(\frac{1700-1400}{200}\right) = 1 - 0.9332 = 0.0668$ (in MATLAB: `normcdf(1700,1400,200,'upper')`)

(v) F — The area under a PDF can be less than or equal to 1. The area under a PDF must always equal 1: $\int_{-\infty}^{\infty} f_X x(dx) = 1$.

Problem 2 *(2 points)*

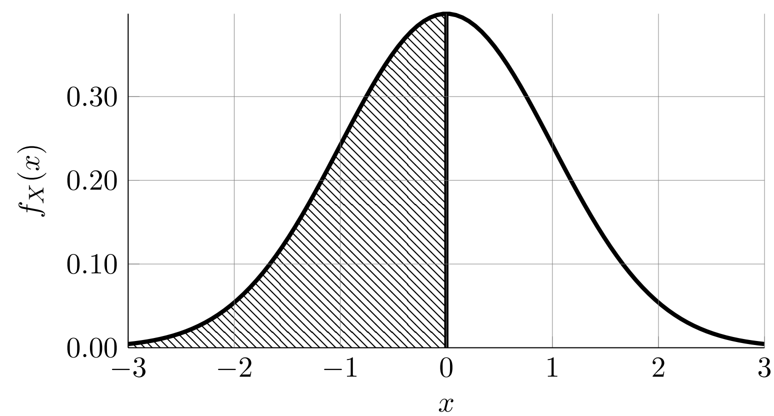
Show brief amount of work for partial credit if answer is wrong. Not required however for full credit.
No penalty if student uses $\geq$ instead of $>$ (and vice versa) or $\leq$ instead of $<$ (and vice versa).

[1] **(a)** Write down the expression of the probability represented by the shaded portion of the normal PDF below. For example, $P(X \leq 2)$. Note that a dashed vertical boundary indicates "$>$" or "$<$," while a solid vertical boundary indicates "$\geq$" or "$\leq$."



Answer: $P(X > 6)$

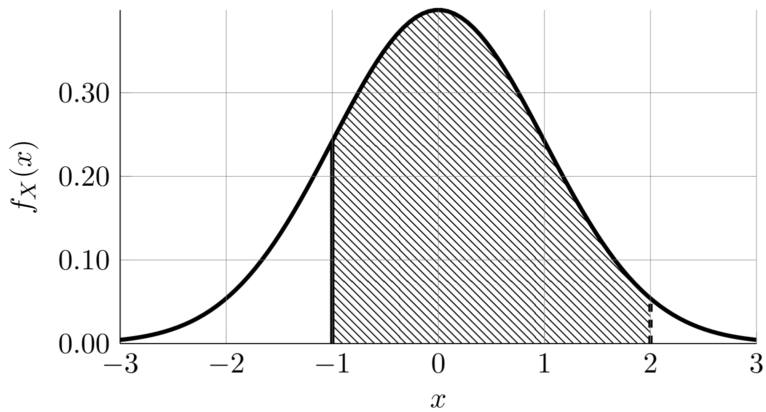
[1] **(b)** Write down the expression of the probability represented by the shaded portion of the normal PDF below. For example, $P(X \leq 2)$. Note that a dashed vertical boundary indicates "$>$" or "$<$," while a solid vertical boundary indicates "$\geq$" or "$\leq$."



Answer: $P(X \leq 0)$

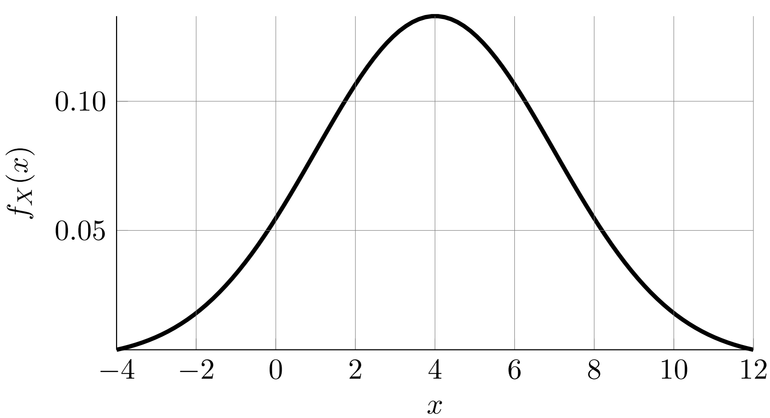
Problem 3 *(2 points)*

(a) Write down the expression of the probability represented by the shaded portion of the normal PDF below. For example, $P(X \leq 2)$. Note that a dashed vertical boundary indicates ">" or "<," while a solid vertical boundary indicates "≥" or "≤." *[1]*



Answer: $P(-1 \leq X < 2)$

(b) Below is the PDF of a given normal distribution. What is the median of this distribution? *[1]*

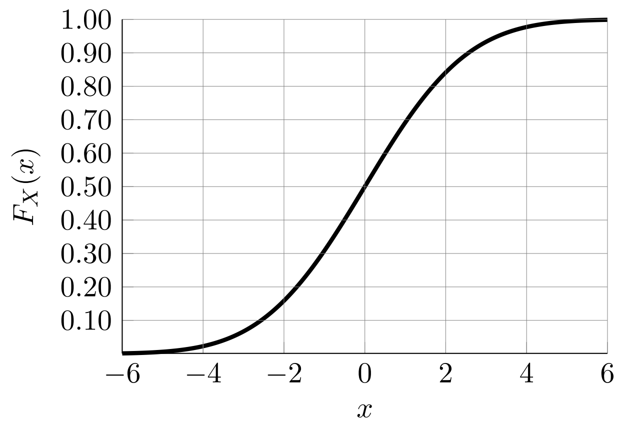


Answer: The median x_m is 4.

Problem 4 *(4 points)*

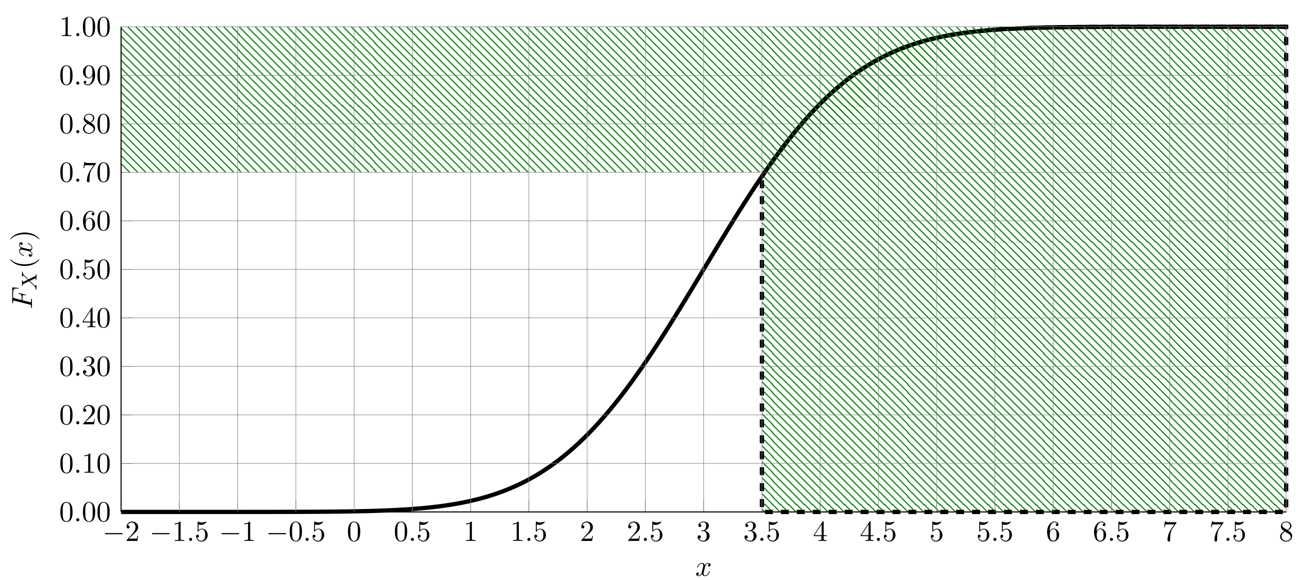
In the following problems, show how you arrive at the answer on the graph.

[2] **(a)** Below is the CDF of a given normal distribution. What is the mean of this distribution?



Answer: The mean $E(X) = 0$.

[2] **(b)** Below is the CDF of a given normal distribution. Estimate the probability $P(X > 3.5)$.

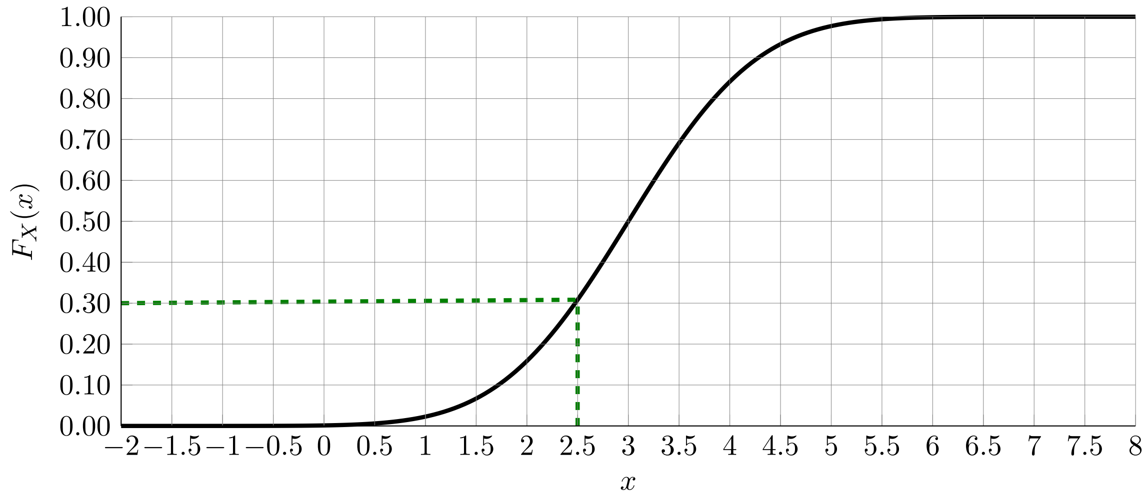


Answer: $P(X > 3.5) \approx 1 - 0.7 = 0.3$

Problem 5 *(4 points)*

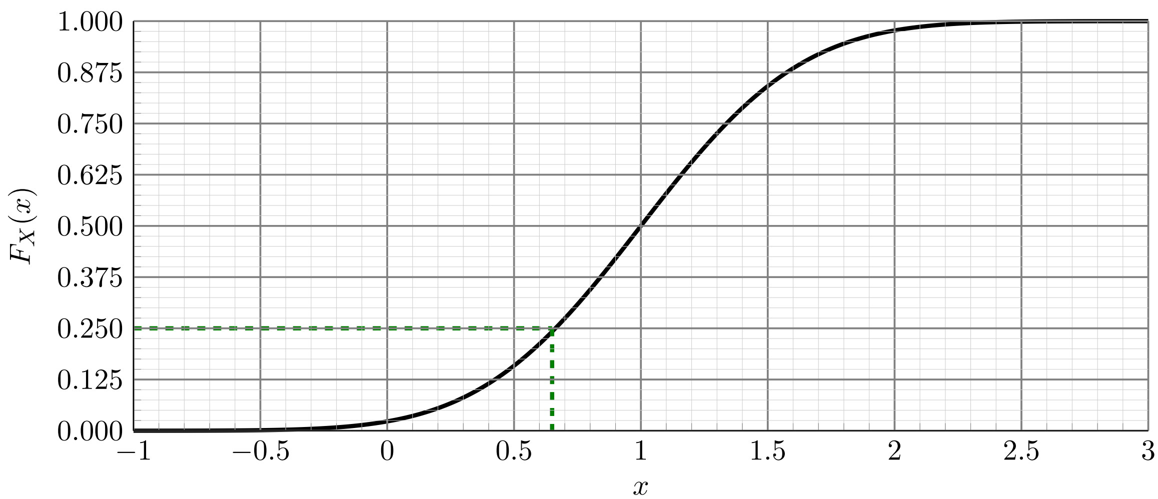
In the following problems, show how you arrive at the answer on the graph.

(a) Below is the CDF of a given normal distribution. Estimate the quantity $F_X^{-1}(0.3)$. *[2]*



Answer: $F_X^{-1}(0.3) \approx 2.5$. In other words, $P(X \leq 2.5) \approx 0.3$.

(b) Below is the CDF of a given normal distribution. Estimate the first quartile of the distribution. *[2]*



Answer: The first quartile $Q1 \approx 0.65$.

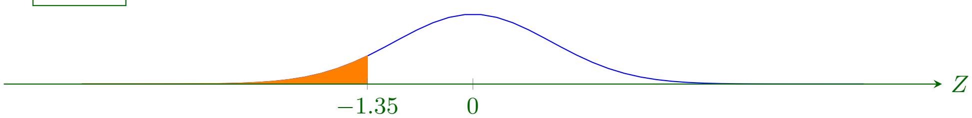
Problem 6 *Standard Normal Distribution (8 points)*

What percent of a standard normal distribution $\mathcal{N}(\mu = 0, \sigma = 1)$ is found in each region? Sketch the accompanying curve along with your answer.

> The key to making good sketches in the following problems is to realize that 99.73% of the normal distribution falls within ± 3 standard deviations from the mean. For the `standard` normal distribution, the mean is 0 and the standard deviation is 1. Thus, much of the curve will lie between -3 and $+3$.

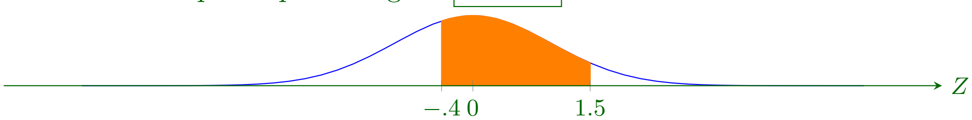
[2 pts] (a) $Z < -1.35$

From tables: $\Phi(-1.35) = 1 - \Phi(1.35) = 1 - 0.9115 = 0.0885$. The required percentage is $\boxed{8.85\%}$.



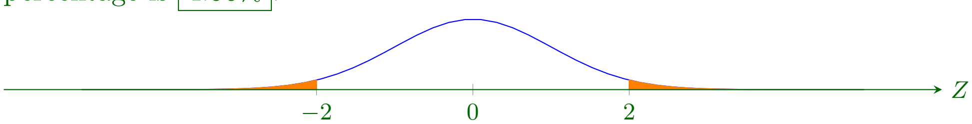
[3 pts] (b) $-0.4 < Z < 1.5$

$P(-0.4 < Z < 1.5) \approx \Phi(1.5) - \Phi(-0.4) = 0.9332 - (1 - \Phi(0.4)) = 0.9332 - 1 + 0.6554 = 0.5886$. The required percentage is $\boxed{58.86\%}$.



[3 pts] (c) $|Z| > 2$

$P(|Z| > 2) = 1 - P(|Z| \leq 2) = 1 - P(-2 < Z \leq 2) \approx 1 - [\Phi(2) - \Phi(-2)] = 1 - [\Phi(2) - (1 - \Phi(2))] = 1 - [2\Phi(2) - 1] = 2 - 2(0.9772) = 0.0456$. The required percentage is $\boxed{4.56\%}$.



Problem 7: Normal Distribution *(5 points)*

The average daily high temperature in June in LA is 77°F with a standard deviation of 5°F. Suppose that the temperatures in June closely follow a normal distribution.

(a) What is the probability of observing an 83°F temperature or higher in LA during a randomly chosen day in June? *[2]*

1. First, compute the Z-score for $X = 83°F$:

$$Z = \frac{X - \mu}{\sigma} = \frac{83 - 77}{5} = 1.2$$

2. Next, using the standard normal distribution table, we find that:

$$\Phi(1.2) \approx 0.8849$$

3. The probability of $X \geq 83°F$ is:

$$P(X \geq 83) = 1 - \Phi(1.2) = 1 - 0.8849 = 0.1151$$

(b) How cool are the coldest 10% of the days (days with lowest average high temperature) during June in LA? *[3]*

1. The Z-score corresponding to the 10th percentile can be found using the inverse of the standard normal CDF:

$$\Phi^{-1}(0.10) = -1.28$$

2. Using the Z-score formula:

$$Z = \frac{X - \mu}{\sigma}$$

3. Substituting the known values:

$$-1.28 = \frac{X - 77}{5}$$

4. Solving for X:

$$X = 77 + (-1.28) \cdot 5 = 77 - 6.4 = 70.6°F$$