Problem Set 3

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CEE 260/MIE 273: Probability & Statistics in Civil Engineering *09.16.2025*

Due September 23, 2025 at 1:00 PM as PDF and .ipynb/.m files uploaded on Canvas. If it helps and if possible, you can write your responses directly on this document and upload it instead. ***Show as much work as possible in order to get FULL credit.*** *There are 4 problems with a total of 31 points available.*

Problem 1 *(8 points)*

Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

(i) ☐ Given three events D, A and G. If $P(\overline{D}|AG) = 1$, then $P(D)$ is an impossible event.

(ii) ☐ Two events A and B can both be mutually exclusive and yet collectively exhaustive.

(iii) ☐ If $P(A|B) = P(A)$ for a set of events A and B, then both events are dependent.

(iv) ☐ Events E and F are mutually exclusive and collectively exhaustive. If $P(E) = 0.3$, then $P(F) = 0.7$.

(v) ☐ Events E and F are collectively exhaustive but *not* mutually exclusive. For a given event A, its probability can be obtained by $P(A) = P(AE) + P(AF)$.

(vi) ☐ The number of ways 6 books can be arranged on a bookshelf is 720. If two of the books are identical, then the total number of distinct arrangements is 360.

(vii) ☐ The number of distinct subgroups of size n that can be formed from a larger group of m objects is given by $\frac{m!}{n!(n-m)!}$.

(viii) ☐ The events E_1 and E_2 are independent. If $P(E_1) = 0.4$ and $P(E_1E_2) = 0.04$, then $P(E_2) = 0.1$.

Problem 2 *(8 points)*

Data collected at elementary schools in DeKalb County, GA, suggest that each year roughly 25% of students miss exactly one day of school, 15% miss 2 days, and 30% miss 3 or more days due to sickness.

Example

Let X be the number of school days missed in a year. The probability that a student chosen at random misses 2 or more days of school in a year is given by:

$$P(X \geq 2) = P(X = 2) + P(X \geq 3) = 0.15 + 0.30 = 0.45$$

(a) What is the probability that a student chosen at random does not miss any days of school due to sickness this year?

(b) What is the probability that a student chosen at random misses no more than one day?

(c) What is the probability that a student chosen at random misses at least one day?

(d) What is the probability that a student chosen at random misses one or two days of school a year?

Problem 3 *Bayes' Theorem (8 points)*

Given an earthquake of intensity

$$X : \{\text{light } (L), \text{moderate } (M), \text{important } (I)\} \tag{0.1}$$

and a structure that can be in a state

$$Y : \{\text{damaged}(D), \text{undamaged}(\overline{D})\} \tag{0.2}$$

The likelihood of damage given earthquake intensity is given by the following conditional probabilities:

$$P(D|L) = 0.01$$
$$P(D|M) = 0.10$$
$$P(D|I) = 0.60$$

and the prior probability of each intensity is given by

$$P(L) = 0.90$$
$$P(M) = 0.08$$
$$P(I) = 0.02$$

(a) Draw and label a Venn diagram illustrating the given events. *[5]*

(b) Find the total probability $P(D)$. *[3]*

Problem 4 *Bayes' Theorem (continued; 7 points)*

Use the quantities provided and the results from Problem to answer the following questions.

[6] **(a)** Use Bayes' Theorem to find the posterior probabilities $P(L|D)$, $P(M|D)$ and $P(I|D)$.

[1] **(b)** Show that the probabilities $P(L|D)$, $P(M|D)$ and $P(I|D)$ sum up to 1.