Problem Set 3 **Prof. Oke**

CEE 260/MIE 273: Probability & Statistics in Civil Engineering *09.16.2025*

Due September 23, 2025 at 1:00 PM as PDF and .ipynb/.m files uploaded on Canvas. If it helps and if possible, you can write your responses directly on this document and upload it instead. ***Show as much work as possible in order to get FULL credit.*** *There are 4 problems with a total of 31 points available.*

Problem 1 *(8 points)*

Respond "T" (*True*) or "F" (*False*) to the following statements. Use the boxes provided. Each response is worth 1 point.

(i) F — Given three events D, A and G. If $P(\overline{D}|AG) = 1$, then $P(D)$ is an impossible event. We only know that $P(D|AB) = 0$, which means that $P(DAB) = 0$. We do not have enough information to determine whether $P(D) = 0$ or not.

(ii) T — Two events A and B can both be mutually exclusive and yet collectively exhaustive.

(iii) F — If $P(A|B) = P(A)$ for a set of events A and B, then both events are dependent.

(iv) T — Events E and F are mutually exclusive and collectively exhaustive. If $P(E) = 0.3$, then $P(F) = 0.7$. True. The probability of the union of collectively exhaustive events is 1.

(v) F — Events E and F are collectively exhaustive but *not* mutually exclusive. For a given event A, its probability can be obtained by $P(A) = P(AE) + P(AF)$. This only holds if E and F are mutually exclusive. But since they are not, then $P(A) = P(AE \cup AF) = P(AE) + P(AF) - P(AE \cap AF)$ (addition rule).

(vi) T — The number of ways 6 books can be arranged on a bookshelf is 720. If two of the books are identical, then the total number of distinct arrangements is 360. The permutations are $\frac{n!}{n_1!}$, where $n_1 = 2$. Thus $\frac{6!}{2} = 360$.

(vii) F — The number of distinct subgroups of size n that can be formed from a larger group of m objects is given by $\frac{m!}{n!(n-m)!}$. (The correct answer is $\frac{m!}{n!(m-n)!}$)

(viii) T — The events E_1 and E_2 are independent. If $P(E_1) = 0.4$ and $P(E_1E_2) = 0.04$, then $P(E_2) = 0.1$. $(P(E_2) = P(E_1E_2)/P(E_1) = 0.04/0.4 = 0.1)$

Problem 2 *(8 points)*

Data collected at elementary schools in DeKalb County, GA, suggest that each year roughly 25% of students miss exactly one day of school, 15% miss 2 days, and 30% miss 3 or more days due to sickness.

Example

Let X be the number of school days missed in a year. The probability that a student chosen at random misses 2 or more days of school in a year is given by:

$$P(X \geq 2) = P(X = 2) + P(X \geq 3) = 0.15 + 0.30 = 0.45$$

(a) What is the probability that a student chosen at random does not miss any days of school due to sickness this year?

Given: $P(X = 1) = 0.25,\ P(X = 2) = 0.15,\ P(X \geq 3) = 0.30$. Then

$$P(X = 0) = 1 - \big(0.25 + 0.15 + 0.30\big) = 1 - 0.70 = \boxed{0.30}.$$

(b) What is the probability that a student chosen at random misses no more than one day?

$$P(X \leq 1) = P(0) + P(1) = 0.30 + 0.25 = \boxed{0.55}.$$

(c) What is the probability that a student chosen at random misses at least one day?

$$P(X \geq 1) = 1 - P(0) = 1 - 0.30 = \boxed{0.70}.$$

(d) What is the probability that a student chosen at random misses one or two days of school a year?

$$P(X = 1 \text{ or } 2) = P(1) + P(2) = 0.25 + 0.15 = \boxed{0.40}.$$

Problem 3 *Bayes' Theorem (8 points)*

Given an earthquake of intensity

$$X : \{\text{light } (L), \text{moderate } (M), \text{important } (I)\} \tag{0.1}$$

and a structure that can be in a state

$$Y : \{\text{damaged}(D), \text{undamaged}(\overline{D})\} \tag{0.2}$$

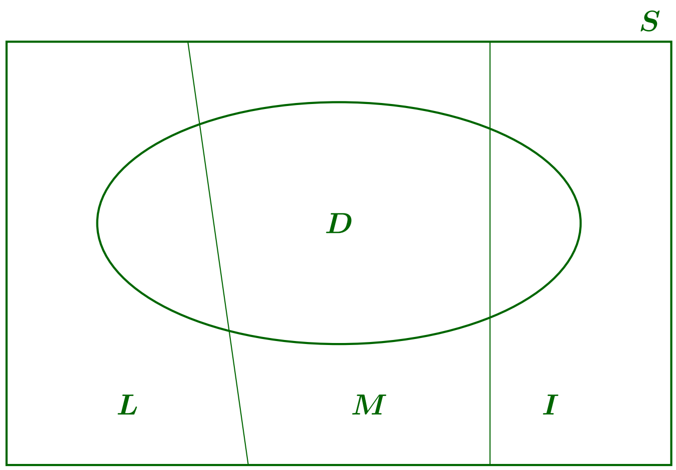
The likelihood of damage given earthquake intensity is given by the following conditional probabilities:

$$\begin{aligned} P(D|L) &= 0.01 \\ P(D|M) &= 0.10 \\ P(D|I) &= 0.60 \end{aligned}$$

and the prior probability of each intensity is given by

$$\begin{aligned} P(L) &= 0.90 \\ P(M) &= 0.08 \\ P(I) &= 0.02 \end{aligned}$$

(a) Draw and label a Venn diagram illustrating the given events. *[5]*



(b) Find the total probability $P(D)$. *[3]*

$$\begin{aligned} P(D) &= P(D|L)P(L) + P(D|M)P(M) + P(D|I)P(I) \\ &= 0.01(0.90) + 0.10(0.08) + 0.60(0.02) \\ &= 0.029 \end{aligned}$$

Problem 4 *Bayes' Theorem (continued; 7 points)*

Use the quantities provided and the results from Problem to answer the following questions.

[6] **(a)** Use Bayes' Theorem to find the posterior probabilities $P(L|D)$, $P(M|D)$ and $P(I|D)$.

$$P(L|D) = \frac{P(D|L)P(L)}{P(D)} = \frac{0.01(0.90)}{0.029} = 0.31$$
$$P(M|D) = \frac{P(D|M)P(M)}{P(D)} = \frac{0.10(0.08)}{0.029} = 0.28$$
$$P(I|D) = \frac{P(D|I)P(I)}{P(D)} = \frac{0.60(0.02)}{0.029} = 0.41$$

[1] **(b)** Show that the probabilities $P(L|D)$, $P(M|D)$ and $P(I|D)$ sum up to 1.

$0.31 + 0.28 + 0.41 = \boxed{1}$.